Motion State Estimation for an Autonomous Vehicle-Trailer System Using Kalman Filtering-based Multisensor Data Fusion

Youngshik Kim
Mechanical Engineering, Hanbat National University, Daejon, 305-719, Korea
youngshik@hanbat.ac.kr

Abstract. In this research we present a Kalman filtering-based motion state estimation method for an autonomous vehicle-trailer system by fusing multiple sensor data, which can be applied directly to autonomous navigation and motion control. The autonomous vehicle-trailer system consists of an autonomous vehicle and a passive trailer which are coupled by a trailer hitch. Our vehicle-trailer system is equipped with the Global Positioning System (GPS), encoder-based odometry, and hitch angle sensors. Using GPS and odometry sensors, we obtain two independent vehicle motion information which includes orientation, position (localization), and velocity data. We then apply a Kalman filtering method by fusing odometry, GPS, and hitch angle sensor data simultaneously to provide robust and more accurate motion state estimation for the vehicle-trailer system in the presence of sensor noises. Finally, we verify our Kalman filtering-based motion state estimation method in simulation and experimental tests.

Keywords: Kalman filter, localization, motion state estimation, multisensor fusion, vehicle-trailer.

1. INTRODUCTION

The subject of this research is motion state estimation for an autonomous vehicle-trailer system in the presence of sensor noises. Fig. 1 shows our target application, the autonomous robotic vehicle-trailer system. The autonomous vehicle-trailer system consists of an autonomous vehicle and a passive trailer which are coupled by a trailer hitch. A fully autonomous or self-driving vehicle is a robot that can move or transport goods and passengers independently without a human driver. More recently, autonomous vehicles or robots have received considerable attention from the general public as well as researchers since the use of these vehicles can improve quality of human life significantly considering safety, mobility, and social costs of driving a car. More specifically, the adoption of autonomous vehicles will be able to save lives by reducing traffic accidents, reduce congestion, pollution, and fuel consumption, alleviate parking problems and human drivers from driving chores, and increase mobility for people who cannot drive (i.e., blind, disabled, intoxicated, and underaged people).
Recently, many commercial automobile manufacturers and research groups have presented autonomous vehicles. Some of these vehicle systems have been demonstrated in urban environments. Several competitions including DARPA Grand Challenges were also held to advance and promote autonomous vehicle technologies. In many countries including the U.S., laws to regulate the use of autonomous vehicles are proposed or passed as autonomous vehicle technologies have advanced significantly.

Furthermore, a passive trailer is popularly used to transport goods or materials. The trailer can be used for personal, recreational, business purposes (i.e., boat, utility, travel, mobile home, livestock, construction, and cargo trailers). Using a trailer hitch, the trailer can be drawn by a powered vehicle such as a car or truck. Towing a trailer with a car is more difficult than simply driving a car. In fact, backing a car-trailer system is more challenging and confusing for an inexperienced human driver because its kinematic motion states are naturally unstable given a continuous velocity input. Thus, the autonomous vehicle-trailer system will provide convenience for a human driver particularly with difficulties in backing and parking a trailer in addition to benefits of autonomous vehicles.

The autonomous robot system should have capabilities of sensing its environment and navigating with a high level of autonomy. In autonomous systems, correct estimation of current motion information is thus a fundamental and important process in making any decision. It is particularly important to accurately estimate motion states of the system such as orientations, positions (localization), and velocities from given limited sensor data to solve navigation and control problems. Motion state information is ultimately used as an input for autonomous navigation and motion control algorithms. Motion state estimation is thus an important research subject for autonomous robot systems.

Multisensor data fusion has drawn many researchers’ interests since multisensory data can improve accuracy and robustness of observations while overcoming physical limitations and inaccuracy of single sensor data. As a result, many sensor fusion methods have been developed based on probabilistic methods and/or statistical theories as discussed in (Durrant-Whyte and Henderson, 2008). These sensor fusion methods include Kalman Filter (KF) (Bishop, 2006), variants of the Kalman filter

![Fig. 1. Autonomous robotic vehicle-trailer system.](image-url)
including the Extended Kalman Filter (EKF) (Crassidis, 2006; Li et al., 2013; Mourikis et al., 2007), sequential Monte Carlo techniques (Ba-Ngu et al., 2005), and Bayesian filter (Fox et al., 2003). Note the standard KF is limited to linear state models such that the EKF is proposed to handle nonlinear state models. In particular, Kalman filtering techniques have been widely used in many applications (Quadri and Sidek, 2013; Quadri and Sidek, 2014) including image processing, signal processing, and vehicle navigation and controls because of the ease of implementation and improved performance.

In robotics, Kalman filtering is popularly used to solve navigation and control problems such as localization, mapping, motion estimation, motion control, and motion planning (Bellotto and Huosheng, 2009; Guo et al., 2010; Jetto et al., 1999; Li et al., 2013; Marin et al., 2014; Pinto et al., 2012; Rezaei and Sengupta, 2007; Sung et al., 2009). In past research (Crassidis, 2006; Guo et al., 2010; Nemra and Aouf, 2010; Rezaei and Sengupta, 2007), a relative position sensor is typically combined with an absolute position sensor for multisensor fusion. Note relative position sensors include odometry and Inertial Measurement Unit (IMU), which uses dead reckoning techniques. In contrast, absolute position sensors contain the Global Positioning System (GPS) and landmark detection sensors. As a result, several different sets of sensor combinations are presented depending on applications and availability. Note each sensor alone is subject to physical limitations and sensor noises. For example, the GPS can provide absolute position information outside the robot without sensor drifts in the absolute reference frame. However, the GPS data may be influenced critically by given external environment.

In this research we propose a Kalman filter-based sensor fusion algorithm to estimate motion states of an actual vehicle-trailer system. We use three different sensors (i.e., odometry, GPS, and hitch angle sensors) to estimate complete motion states of the vehicle-trailer system. In this case, we will consider total eight state variables, two control inputs, and four sensor measurements for the vehicle-trailer system. A vehicle-trailer is widely used to transport objects for diverse personal and professional applications as mentioned. However, less attention was paid to sensor fusion algorithms for autonomous vehicle-trailer systems in previous research. Our main contribution is thus our Kalman filter-based sensor fusion and motion state estimation algorithms considering complete states of the vehicle-trailer system. We also contribute to validate the proposed algorithm in simulation and experiment. Our motion state estimation algorithm can be applied directly to provide an input for navigation and motion control algorithms of autonomous vehicle-trailers.

This paper is organized as follows: A continuous vehicle-trailer model is presented in Sec. 2. In Sec. 3 we discuss Kalman filtering techniques for multisensor data fusion. We also present the discrete-time state-space model for the vehicle-trailer system using the continuous vehicle-trailer model. Simulation and experimental results are presented to validate our Kalman filtering-based motion state estimation method in Sec. 4. Conclusions are finally provided in Sec. 5.
2. Vehicle-trailer Model

In this section, we present a general continuous-time kinematic model for the vehicle-trailer system, which will be applied to formulate the discrete-time linear state-space model for the Kalman filter in the next section. We can derive the general vehicle-trailer model considering steering kinematics of the vehicle-trailer system illustrated in Fig. 2. The system state variables are the Cartesian coordinates, \((x, y)\) at the point \(C_1\), the heading angle, \(\theta\), and the hitch angle, \(\phi\). The state equations for this vehicle-trailer system are then,

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega \\
\dot{\psi} &= v \left[ \frac{\tan \phi}{L} \left( \frac{L_2}{L} \right) - \sin \psi \right],
\end{align*}
\]

where \(v\) and \(\omega\) are, respectively, the linear velocity and angular velocity at the rear axle center, \(C_1\), \(\phi\) is the steering angle, \(L\) is the distance between front and rear axles, \(L_1\) is the hitch length, and \(L_2\) is the trailer length. Note \(v\) and \(\omega\) are control inputs in this model.

Furthermore, using the curvature definition in Fig. 2 and a trigonometric identify for the right-angled triangle, \(OC_0C_1\), we can express the curvature at the point \(C_1\) by,

\[
\kappa = \frac{1}{OC_1} = \frac{\dot{\theta}}{v} = \frac{\tan \phi}{L}.
\]

As a result, we can rewrite our state model for the vehicle-trailer system applying (2) to (1) in vector-matrix form by,

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\frac{L_2}{L_2 + L_1 \cos \psi} & - \frac{L_1 L_2 \cos \psi}{L_2} & \frac{L_1}{L} & \frac{L_1 L_2 \sin \psi}{L_2}
\end{bmatrix}
\begin{bmatrix}
v \\
\omega
\end{bmatrix}.
\]

3. Kalman Filtering

We now discuss general Kalman filtering techniques for a nonlinear system, which can also be readily applied to linear systems. In this case, using the nonlinear system state equations (1) or (3), we can easily formulate the following nonlinear state model and observation model in vector-matrix form by,
where $x$ is the state vector and $u$ is the known control input vector, and $z$ is the observation vector simply assuming an ideal model. This model (4) can then be applied to the EKF algorithm. Process and sensor measurement noises will be considered later in the prediction and measurement update equations by applying noise covariance matrices.

The continuous system state model in (4) is then transformed for discrete time step $k$ and the sampling time, $\Delta t$, since Kalman filtering techniques are established in discrete time. As a result, the discrete-time state model at time step $k$ can be expressed by,

$$
\boldsymbol{x}^k = f(\boldsymbol{x}^{k-1}, \boldsymbol{u}^k, 0) = A(k)\boldsymbol{x}^{k-1} + B(k)\boldsymbol{u}^k,
$$

which can be explicitly determined by,

\[
\begin{bmatrix}
    x^k \\
    y^k \\
    \theta^k \\
    \dot{x}^k \\
    \dot{y}^k \\
    \dot{\theta}^k \\
    \psi^k \\
\end{bmatrix} =
\begin{bmatrix}
    x^{k-1} + v^k \cos \theta^{k-1} \Delta t \\
    y^{k-1} + v^k \sin \theta^{k-1} \Delta t \\
    \theta^{k-1} + \omega^k \Delta t \\
    v^k \cos \theta^{k-1} \\
    v^k \sin \theta^{k-1} \\
    \omega^k \\
    \psi^{k-1} + \left(-v^k \sin \psi^{k-1} / L_2 + \omega^k \left(L_2 + L_1 \cos \psi^{k-1}\right) / L_2 \right) \Delta t \\
    -v^k \sin \psi^{k-1} / L_2 + \omega^k \left(L_2 + L_1 \cos \psi^{k-1}\right) / L_2 \\
\end{bmatrix},
\]
Note the superscript \( k \) is used to indicate discrete time step \( k \). Also note \( A(k) \) is the transition matrix and \( B(k) \) is the control matrix, which can be computed by linearizing (6) at current time step \( k \). Similarly, we can also describe the discrete observation model given discrete time step \( k \) by,

\[
z^k = h(x^k(t), 0, 0) = H(k)x^k,
\]

where \( H(k) \) is the observation matrix, which maps the sensor measurements to the states at time step \( k \). Likewise, \( H(k) \) can also be obtained by linearizing the observation model in (4). For simpler notations, we will drop \( k \) in matrices \( A, B \), and \( H \) for the rest of this paper.

The Kalman filtering algorithm critically depends on the system states, \( x \), and the state-estimate error covariance, \( P \). This covariance matrix describes the uncertainty associated with the KF/EKF estimates and the correlations among the states. The system states are updated by applying the following time update and measurement update equations recursively over time. Furthermore, process and sensor measurements noises are implemented in the KF/EKF by using the process noise covariance matrix \( Q \) and the measurement noise covariance matrix \( R \), which are assumed to be uncorrelated zero-mean Gaussian white noises.

**Prediction (time update) equations:**

A prediction \( \hat{x}^k \) of the state at time step \( k \) and its predicted error covariance \( \hat{P}^k \) are determined by,

\[
\hat{x}^k = f(\hat{x}^{k-1}, B u^k, 0) = A \hat{x}^{k-1} + B u^k
\]

\[
\hat{P}^k = A \hat{P}^{k-1} A^T + Q
\]

**Measurement update (estimate update) equations:**

At time \( k \) the Kalman gain \( K^k \) is determined. The estimate \( \hat{x}^k \) is then updated by using this Kalman gain, state prediction \( \hat{x}^k \), and observation \( z^k \) as follows,

\[
K^k = \hat{P}^k H^T (H \hat{P}^k H^T + R)^{-1}
\]

\[
\hat{x}^k = \hat{x}^k + K^k (z^k - H \hat{x}^k)
\]

\[
P^k = (I - K^k H) \hat{P}^k
\]

Note the caret superscript in \( \hat{x}^k \) indicates an estimate of the state and the tilde superscript in \( \hat{x}^k \) and \( \hat{P}^k \) indicates a prediction at current time step \( k \).

Moreover, we can treat our nonlinear state model (6) as a linear model for Kalman filtering by simply choosing \( v \) and \( \omega \) as the control inputs similar to the continuous-time state equations (3). In our KF implementation, we define eight system state variables, two control inputs, and four sensor measurements such that we have,
\[ x = \begin{bmatrix} x & y & \theta & \hat{x} & \hat{y} & \psi \end{bmatrix}^T \]
\[ z = \begin{bmatrix} x & y & \theta & \psi \end{bmatrix}^T \]
\[ u = \begin{bmatrix} v & \omega \end{bmatrix}^T \]

Note all these variables (positions, orientations, and velocities) are associated with motion estimates or measurements of the vehicle-trailer system in the global Cartesian reference frame. Using the standard KF, the transition, observation, and control matrices are then expressed by,

\[
A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
B = \begin{bmatrix} \cos \theta^{k-1} \Delta t & 0 \\ \sin \theta^{k-1} \Delta t & 0 \\ 0 & \Delta t \\ \cos \theta^{k-1} & 0 \\ \sin \theta^{k-1} & 0 \\ 0 & 1 \\ \sin \psi^{k-1} \Delta t / L_z & \omega \psi^{k-1} \left( L_z + L_1 \cos \psi^{k-1} \right) \Delta t / L_z \\ \sin \psi^{k-1} / L_z & \omega \psi^{k-1} \left( L_2 + L_1 \cos \psi^{k-1} \right) / L_z \end{bmatrix},
\]
\[
H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.
\]

In this case, the measurement noise covariance \( R \) and the control input noise covariance \( R_v \) are determined using diagonal matrices,

\[
R = \text{diag}\left[ \sigma_x^2, \sigma_y^2, \sigma_\theta^2, \sigma_{\hat{x}}^2, \sigma_{\hat{y}}^2, \sigma_\psi^2 \right], \quad R_v = \text{diag}\left[ \sigma_v^2, \sigma_\omega^2 \right].
\]

where standard deviations, \( \sigma \), for measurement and control input noises can be evaluated from GPS and odometry sensors such that we have \( \sigma_x = 0.0124, \sigma_y = 0.0037, \sigma_\theta = 0.034, \sigma_{\hat{x}} = 0.034, \sigma_{\hat{y}} = 0.0149 \), and \( \sigma_\psi = 0.1 \) for our experiments. Furthermore, we may assume the process noise is mainly caused by noises in the control inputs observing the model (5) without loss of generality. The process noise covariance \( Q \) is then computed by,

\[
Q = BR_v B^T.
\]
4. Validation

In this section we evaluate KF-based multisensor data fusion for motion state estimation in simulated and experimental tests. Our test results are presented and then discussed briefly.

4.1. Simulation

We first evaluate our KF-based motion state estimation method in simulation. We apply a smooth path trajectory for the vehicle-trailer system. This path is generated by using control inputs, \( v \) and \( \omega \). As discussed in Sec. 3, we use covariance matrices composed of noise standard deviations to consider process and measurement noises in the KF assuming independent Gaussian white noises. In particular, we apply larger noise standard deviations, \( \sigma_x=0.5 \) and \( \sigma_y=0.5 \), for the GPS sensor to demonstrate our estimation method in a worst scenario. We also adopt noisy control inputs to consider

![Fig. 3. Path trajectory from simulation.](image)

![Fig. 4. Applied control inputs from simulation.](image)
odometry sensor noises as shown in Fig. 4. We use the following vehicle parameters, $L_1=1.403$ m and $L_2=2.767$, for the actual vehicle-trailer system. Fig. 3 shows simulated trajectory paths of vehicle-trailer system. Note path estimates from the KF-based sensor fusion are smooth and almost identical to ideal paths. These results indicate our estimation method provides robust and accurate state estimates by rejecting sensor noises in orientation, position, and velocity measurements efficiently as desired.

4.2. Experiment

We evaluate our KF-based motion state estimation method using the actual vehicle-trailer system, Fig. 1. The potentiometer-based hitch angle sensor is mounted above a hitch ball and coupler. Quadrature encoders are installed on the rear wheels for

![Fig. 5. Path trajectory estimation from experiment.](image)

![Fig. 6. Calculated or estimated velocities from experiment.](image)
odometry. We drive the vehicle forward for a distance to obtain actual measurement and control input data. Fig. 5 shows respective trajectory paths of the vehicle and trailer. These vehicle paths are estimated from the KF-based method (Kalman estimate), GPS, odometry (Odo.), and combination of the GPS and odometry (GPS+Odo.), respectively.

Control inputs observed from odometry are similar to estimated or calculated velocities in our KF-base estimation results as shown in Fig. 6. However, position estimates from odometry shows larger drift because the heading angle estimated from odometry is relatively smaller as illustrated in Fig. 7. This smaller heading angle estimation then leads to considerable errors in linear velocity estimates in the x and y directions. Furthermore, our experimental results show that we can decrease this drift significantly simply combining odometry and GPS data. Moreover, the KF-based estimation method can provide robust estimation results in spite of some initial state errors applying positive definite initial state-estimate error covariance, $P(0)$. Most importantly, we experimentally verify that our KF-based method provides robust and well-converging estimation results while rejecting noises effectively as designed.

5. Conclusions

In this research we propose a KF-based motion state estimation method for an autonomous vehicle-trailer system using multisensor data fusion. We use simulation
to validate the proposed estimation method in a worst case scenario with larger GPS sensor noises. We then validate the proposed estimation method using position, heading angle, hitch angle, and velocity data experimentally measured from the actual vehicle-trailer system. Our simulation and experimental results verify we can estimate motion states of the vehicle-trailer system with robustness in the presence of relatively larger sensor noises using the proposed KF-based multisensor fusion. We also find even a simple combination of GPS and odometry data can improve position and velocity estimates significantly rather than the use of single sensor data.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (2012R1A1A1011457). The author would also like to thank Dr. Minor for his inspiration and support in previous study.

Literature Cited


Bishop, G.W.a.G., 2006. An Introduction to the Kalman Filter. Technical Report TR 95-041, Department of Computer Science, University of North Carolina at Chapel Hill, Chapel Hill, NC, USA.,


